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# Scaling Challenges in Explanatory Reasoning

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## Abstract

Despite well-known limitations, human cognition exhibits remarkable abilities for scaling to factors like task complexity and knowledge base size. In this paper, we revisit a recently proposed theory of explanatory inference and its implementation in the PENUMBRA system, which we hypothesize will support similar properties. We examine – analytically and empirically – the computational costs associated with the architecture's basic inference cycle, which alternates between selecting a focus belief, elaborating current explanations, and repairing violated constraints. At a higher level, we study PENUMBRA's effectiveness at searching the space of alternative explanations for a set of observations. We conclude with comments on related work and proposals for future research.

## 1. Introduction

Decades of research have established that humans are inherently limited information processors. People have imperfect memories, they can focus on only one thing at a time, and they have difficulty with long chains of reasoning. Yet despite these limitations, they are remarkably effective at many complicated cognitive tasks, and their performance improves further as they gain experience in an area, with experts outperforming novices substantially in terms of both speed of processing and quality of results. Any complete computational theory of high-level intelligence must account for these intriguing and important phenomena. Human cognition shows an amazing ability to scale well to both the complexity of tasks and the size of knowledge bases, due at least partially to its reliance on heuristics to mitigate its processing constraints.

In this paper, we address the general task of explaining observations, in which an intelligent system, human or otherwise, attempts to understand events it encounters in terms of available knowledge. Many instances of such explanation involve abductive reasoning (Peirce, 1878), in that they require one to introduce plausible assumptions. There is a substantial literature on abductive and explanatory inference that we will not review here. Some of this tradition has examined issues of efficiency, but there have been few efforts to draw links to human cognition. For example, we know that, as people acquire domain expertise, their increased knowledge does not slow down their processing. Often they can also construct complex, highly interconnected explanations without much difficulty. Moreover, when confronted with many possible explanations (e.g., different parses for a sentence), people find the best alternative as rapidly as when there are fewer choices (Carroll, 2008).

In the sections that follow, we present a promising account of such scalable explanatory reasoning. We start by reviewing an earlier theory of associative abduction and its implementation in PENUMBRA, an architecture for plausible inference. After this, we examine the computational costs of the system's basic cognitive cycle, both formally and empirically, in terms of factors like the number of rules and their complexity. Next we consider the effort required to construct complete explanations, focusing specifically on scaling to increasing numbers of consistent alternatives. In closing, we discuss related work on the efficiency of complex cognitive tasks and proposals for future research on this important topic.

## 2. A Theory of Explanatory Inference

Consider an everyday example of explanatory reasoning. We hear that Abe has some cash and Bob possesses an automobile, but that later Abe possesses the same car. Even though we did not observe any interaction, we might reasonably assume that one of two transactions took place. Abe may have used his money to buy the car from Bob, but it is also possible that Abe threatened Bob and stole the car. We know that purchases and robbery are two distinct ways to transfer possession of objects, so these two accounts are mutually exclusive. To reason about the situation effectively, we must not only make plausible assumptions about unobserved events, but also keep the competing explanations separate. If we later hear that Abe had actually given his money to Bob, then we would abandon the theft account as inconsistent with the available facts and conclude that an automobile purchase had indeed occurred.

More complex scenarios would involve multi-step inference chains through hierarchical structures, but this simple example clarifies some key points about human reasoning. People are able to explain observations by connecting them through available knowledge, introduce plausible assumptions about relations or events that are not directly observed, process observations incrementally and incorporate them into existing explanations, detect and address conflicting beliefs that keep them from being consistent, and generate alternative accounts when more than one is plausible. In a recent article, Langley and Meadows (2019) presented a theory of explanatory inference that addresses these abilities. They distinguished between *derivational* abduction, in which observed facts serve as the roots of proof graphs, and *associative* abduction, in which they are terminal nodes of such graphs, along with assumptions. The theory falls into the second paradigm and incorporated a number of postulates about representation and processing, which we review in this section. We also summarize PENUMBRA, an implemented architecture that reflects these theoretical ideas. Additional details about the research are available in the earlier article.

### 2.1 Representational Postulates

Langley and Meadows' account distinguishes between stable knowledge and dynamic beliefs, much as other theories of complex cognition (Langley, Laird, & Rogers, 2009), The framework posits two forms of generic knowledge structures. A *definition* specifies a higher-level relation in terms of other more basic ones; these are often organized hierarchically, much as in a logic program or context-free grammar, with defined terms serving as nonterminal symbols. In our example, there would be high-level definitions for purchasing and robbery, along with low-level rules for transferring property. A *constraint* comprises a set of relations that are mutually exclusive, so they indicate inconsistency when they are satisfied jointly. The main constraint in the example is that one cannot both buy and steal an item. A separate, dynamic working memory contains three types of structures that differ in their origins. *Observed* beliefs come from the external environment, *abduced* beliefs are introduced as assumptions, and *derived* beliefs are deduced from other beliefs using knowledge. In the example, some beliefs about possession are observed, others are abduced from definitional rules, and the beliefs about buying and stealing the car are derived.

The theory also states that *justifications* – instances of applied definitions – are stored and organized into higher-level explanations. The latter structures take the form of proof graphs, with observed and abduced beliefs appearing as terminal nodes and with beliefs that are derived from them serving as nonterminals. A classic case involves parse trees, in which observed words are terminal nodes and nonterminal symbols are derived, with different parses having distinct justifications. Moreover, although beliefs are stored in a single working memory, each of them is also associated with one or more distinct *worlds*. These are stored in a distributed manner by annotating each belief with the worlds in which it does *not* hold, so that a given justification can contribute to competing accounts. For instance, two parses of a given sentence typically share many subtrees, each of them associated with multiple worlds. Finally, the framework organizes worlds into a *phylogenetic tree* that traces their evolution, with *closed* worlds containing known constraint violations and *active* ones, at the frontier, thought to be internally consistent.

### 2.2 Processing Postulates

This theory of explanatory reasoning also includes tenets about processing. The most basic claim is that this mental activity involves two cognitive cycles: an outer *observation* loop that accepts new facts from the environment and an inner *inference* loop that extends and revises explanations of these facts. The inference cycle alternates among selecting an existing belief on which to *focus attention*, invoking definitional rules to *elaborate explanations*, and using constraints to *detect and repair inconsistencies* in these accounts. The chosen focus belief mediates retrieval of relevant knowledge, in that processing only considers definitions and constraints with an antecedent which unifies with it. When applying a retrieved definition, the elaboration process generates a derived belief for its head and creates abduced beliefs for any unmatched antecedents. Thus, explanations are constructed from the bottom up, as inference introduces higher level derivations.

In contrast, if retrieval reveals that a constraint is violated – because incompatible beliefs reside in the same world – then a repair process deactivates this inconsistent world and generates new, active children in which the conflict does not arise. The repair mechanism examines the justifications that underlie the incompatible relations, separates all beliefs that support one but not the other into two consistent sets, and then uses them to populate revised worlds. In the prior example, when the buying and stealing interpretations come into competition, this would spawn two worlds, one in which Abe retains his money after stealing the car and another in which Bob possesses they money after selling the vehicle. Elaboration through definitional rules is a monotonic process, whereas the repair of violated constraints is a nonmonotonic activity.

At a higher level, the process of explanation construction is aided by knowledge but driven by data, in that it responds to observations that arrive incrementally. Because multiple accounts of the same observed facts may be possible, this involves a search through a space of alternative explanations that are consistent with the data. Heuristics for selecting focus beliefs, definitions, and constraints determine the order in which candidates are generated, and thus guide the search process. However, because worlds are encoded in a distributed manner, this activity also exhibits a form of implicit parallelism, in that each inference step can elaborate or repair multiple worlds that share the beliefs involved. The earlier article describes these mechanisms in detail, along with the similarity to, and difference from, earlier approaches to other reasoning paradigms.

### 2.3 The PENUMBRA System

Langley and Meadows (2019) also reported PENUMBRA, an implemented system that embodies these theoretical ideas. This comes with a programming language for specifying definitions, constraints, and beliefs that follow the representational postulates mentioned earlier. PENUMBRA's syntax is similar to Prolog (Clocksin & Mellish, 1981), with definitions having a head and antecedents, each comprising a predicate and arguments, with the latter being variables that may be shared across relations. Constraints include a head and a set of antecedents, each having a predicate and arguments, that are mutually exclusive and thus should never appear in the same explanation. Observed, derived, and abduced beliefs are analogous to ground facts in Prolog, with arguments of predicates being constant terms or Skolems. The language does not include a notation for specifying worlds, as processing always starts with a root world and new ones are generated automatically.

The architecture also includes an interpreter, implemented in Steel Bank Common Lisp, for running programs stated in this syntax. This follows the theory by incorporating an observation cycle that accepts external input and an inner inference cycle that constructs explanations. On each inference cycle, PENUMBRA selects a focus belief, checks for violations of constraints linked to this focus, and, if it detects any, deactivates the inconsistent worlds and creates active children that eliminate the problem. If the system detects no violations, it selects a definition with an antecedent that unifies with the focus, then applies the rule to elaborate worlds in which the matched conditions hold. This produces a derived belief based on the rule's head and abduced beliefs based on its unmatched antecedents. Both cycles continue until no more observations are available and no focus belief is selected. PENUMBRA includes eight parameters that determine selection of focus beliefs, constraints, and definitions, with different settings producing different search for explanations.

Langley and Meadows presented evidence that the system behaves as desired in two arenas that involve explanatory reasoning. The first dealt with plan understanding: inference of an agent's intentions from its observed behavior and knowledge about activities. The second addressed the construction of parse trees for given sentences based on knowledge about syntax. Demonstration runs showed that PENUMBRA can construct plausible explanations in these domains, process observations incrementally and incorporate them into existing accounts, introduce default assumptions when needed, detect inconsistencies and revise beliefs in response, and track competing accounts as new evidence becomes available. These results were promising, but they failed to address an issue important to any multi-step reasoning system: its ability to handle *complexity*. In the remaining sections, we combine formal analysis and controlled experiments to tackle this matter.

*Table 1.* Variables that denote (a) processing times for different stages of PENUMBRA's inference cycle and factors that influence them, including (b) those related to total counts and (c) those involving average counts. The cost term $T_D$ should be constant throughout a run because the knowledge base is fixed, whereas $T_F$ should increase during a run as the number of beliefs grows.

| | |
|---|---|
| (a) | $T_F$ is the processing time per cycle to select a focus |
| | $T_C$ is the processing time per cycle to check constraints |
| | $T_D$ is the processing time per cycle to select a definition |
| (b) | $N_R$ is the number of distinct relational predicates |
| | $N_C$ is the number of constraints |
| | $N_D$ is the number of definitions |
| | $N_B$ is the total number of beliefs |
| (c) | $A_C$ is the number of alternatives per constraint |
| | $C_P$ is the number of constraints per predicate |
| | $A_D$ is the number of antecedents per definition |
| | $D_P$ is the number of definitions per predicate |
| | $B_P$ is the number of beliefs per predicate |

## 3. Scalability of the Inference Cycle

Like many cognitive systems, PENUMBRA operates in discrete cycles, with the main work occurring during the inference loop. If we want to ensure that its construction of explanations is efficient, then we must first show that its cycle-level processing scales well to complicating factors. This was the motivation for early research on efficient matching in production system architectures, which led to successful techniques like Rete (Forgy, 1982) and TREAT (Miranker, 1987). In this section, we examine analogous issues for our explanatory reasoning architecture, which we can partition into the computational expenses of selecting focus beliefs, checking for violated constraints, and selecting definitional rules. We start with a formal analysis and then report empirical studies for selecting among definitions, the more expensive activity.

We are interested in components of PENUMBRA's processing time per cycle and the factors that influence them. Table 1 (a) shows the three dependent variables: the time per cycle to select focus, $T_F$; the time per cycle to check constraints, $T_C$; and the time per cycle to select a definitional rule, $T_D$. We can divide the independent factors into two broad groups. One set (b) concerns the *total numbers* of different types of structures, such as: the number of relational predicates, $N_R$; the number of constraints, $N_C$; the number of definitions, $N_D$; and the number of beliefs, $N_B$. Another set of influences (c) involve *relations between* two types of structures, including: the number of alternatives per constraint, $A_C$; the number of constraints per predicate, $C_P$; the number of antecedents per definition, $A_D$; the number of definitions per predicate, $D_P$; and the number of beliefs per predicate, $B_P$. Because PENUMBRA adopts a distributed representation, the number of beliefs per world is also relevant to processing time, but we will not consider it here.

### 3.1 Processing Costs of the Inference Cycle

We can analyze separately the three stages of PENUMBRA's inference cycle – choosing a focus belief, checking for constraint violations, and selecting a definition to apply. The computational cost of the first stage is most straightforward. The time for focus selection should be affected only by the number of beliefs $N_B$ in working memory. For a naive strategy, this gives

$$T_F = j \cdot N_B \ \ ,$$

but we must add a few caveats. One is that $N_B$ will grow over time, as the system applies definitions and elaborates its explanations. However, if alternative worlds share many of their beliefs, as we have observed in practice, this growth will be sublinear in the number of consistent accounts. Also, we can limit the number of beliefs considered, say by ignoring all but the $k$ most recently added elements. Older beliefs would still be available for matching, but they would not compete for attention unless they are refreshed, say when they are inferred again.

The computational expense of constraint checking is somewhat more complicated. Because PENUMBRA indexes constraints by predicates that appear in them, and because it checks them only after it has selected a focus, it must consider only $C_P$ candidate rules. The total number of constraints, $N_C$, should not affect processing time. Each of the retrieved constraints has $A_C$ mutually exclusive antecedents, one of which has already matched the focus belief, so the system must consider the remaining $A_C - 1$ of them. Each of these, in turn, can potentially unify with $B_P$ beliefs that share their predicate, which gives the expression

$$T_C = i \cdot C_P \cdot (A_C - 1) \cdot B_P \ \ .$$

Because the average number of mutually exclusive relations per constraint, $A_C$, will typically be small, the important factors are $C_P$ and $B_P$. Note that, on a given cycle, PENUMBRA may well overlook some related constraints because they do not involve the focus belief, but it may detect them later, when its attention shifts. We have not discussed the cost of repairing violated constraints once detected because the system stores the rule instances that produced each belief, so it can remove the sources of an inconsistency by a simple retrieval process.

Finally, we can analyze PENUMBRA's retrieval and matching mechanisms to calculate the cost of selecting a definitional rule instance on a given cycle. Again, we can assume the system has already selected a focus belief that involves some predicate. Because definitions are indexed by predicates that occur in their antecedents, it will only retrieve the $D_P$ rules in which they appear; the total number of definitions, $N_D$, will not affect processing time. The cost of matching each retrieved rule is influenced by the average number of beliefs, $B_P$, that contain the focus predicate, and the average number of antecedents, $A_D$, in the definition. Because PENUMBRA allows partial matching in support of abductive reasoning, it considers all ways that antecedents (other than the one that unifies with the focus) can match or fail to match.

For each antecedent, it retrieves on average $B_P$ beliefs with the same predicate. The system considers all combinations of these belief-antecedent pairs, as well as the possibility that each antecedent has no match, giving $(B_P + 1)^{(A_D - 1)}$ candidate partial matches. It will reject many candidates because their variables do not bind consistently, but it must still consider them. Multiplying this expression by the number of retrieved definitions gives the equation

$$T_D = k \cdot D_P \cdot (B_P + 1)^{(A_D - 1)}$$

as the computation time required to find all partial matches of definitions that unify with a given focus belief. The constant $k$ includes the time needed per cognitive cycle to evaluate each of these candidates and to select the best-scoring one. This analysis assumes that the factors $D_P$, $A_D$, and $B_P$ remain constant for a particular run, when in fact the last term will vary, but the equation offers a reasonable approximation with implications for PENUMBRA's scaling behavior.

### 3.2 Experimental Studies of Rule Selection

The analyses above suggest a number of hypotheses that are subject to empirical test. We decided to focus on one subprocess, selecting a definition to use in elaboration, because it is substantially more expensive than choosing a focus or checking constraints. The analytical equation for $T_D$ includes three factors, so we designed and carried out controlled experiments that varied each of them separately, along with a fourth important variable, the number of definitions, $N_D$. We devised a set of synthetic knowledge bases and working memories that would let us vary these factors systematically, and thus check the analysis' predictions experimentally.

Table 2 presents abstract versions of the rules for six different values of $D_P$ and $A_D$. Each definition's antecedent takes two arguments, at least one of which is shared with another condition, to ensure relevance to the relational character of many explanatory tasks. For instance, the first rule for the $A_D = 2, D_P = 2$ condition is *(d1 ?x ?y ?z)* ← *(p1 ?x ?y) (p2 ?y ?z)*, which matches against beliefs like *(p1 a b)* and *(p2 b c)*. The table does not show beliefs, which we can vary independently of the definitions by creating multiple copies of element sets like those above that have disjoint arguments. Note that the rules are not organized in a hierarchy, such as that for a context-free grammar; they specify a flat set of connected relations. However, this structure should not affect the results of experiments, which should depend only on the factors from the analysis.

We designed and ran four experiments using these synthetic knowledge bases. For each experimental condition, we ran PENUMBRA 30 times in nonincremental mode, in that all observed beliefs were available at the outset, letting it continue until the system made no further inferences. The number of inference steps in these ranged from 18 to 144. The knowledge bases did not include any constraints, which meant that the only processes in operation were focus and definition selection. Also, we specified that PENUMBRA should only apply rules when all their antecedents were satisfied, so it never introduced any default assumptions, although the mechanism still considered partial matches. For each run, we measured $T_D$, the time to select a definition to apply, in CPU seconds. We averaged the observed values for $T_D$ across cycles within each run and across runs for each condition, which lets us calculate both means and standard errors.

The first and most important conjecture deals with the implication, according to the $T_D$ equation, that the number of definitions does not affect the time needed to select a definitional rule:

- *Processing time per rule selection $T_D$ is independent of the number of definitions $N_D$.*

To test this claim we varied the number of definitions while holding other variables constant ($A_D$ = 3, $D_P$ = 3, $B_P$ = 3). We started from six definitions ($N_D = 6$) and introduced new rules with the same structure but different predicates for other conditions ($N_D = 12, 18, 24, 36, 48$). Figure 1

*Table 2.* Synthetic knowledge structures that vary two factors – the number of antecedents per definition, $A_D$ and the number of definitions per predicate, $D_P$, that should influence the cost of rule selection. Each antecedent takes two arguments, at least one of which it shares with another condition. The parameter $D_P$ ranged from 2 to 10 and $A_D$ ranged from 2 to 8, but the table shows only the simplest knowledge bases.

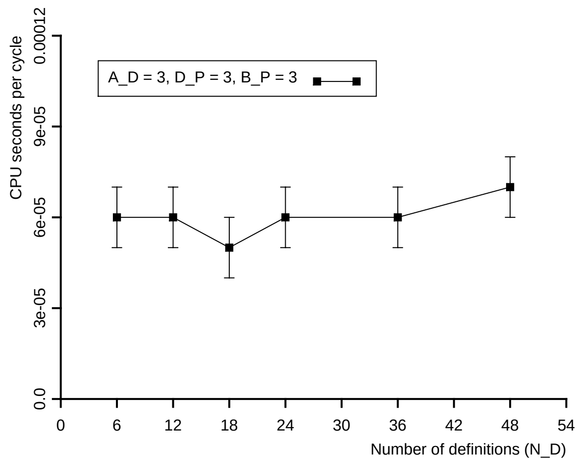
| | $A_D = 2$ | $A_D = 4$ | $A_D = 6$ |
|---|---|---|---|
| $D_P = 2$ | D1 ← P1 P2<br>D2 ← P2 P3<br>D3 ← P3 P4<br>D4 ← P4 P5<br>D5 ← P5 P6<br>D6 ← P6 P1 | D1 ← P1 P2 Q1 Q2<br>D2 ← P2 P3 Q2 Q3<br>D3 ← P3 P4 Q3 Q4<br>D4 ← P4 P5 Q4 Q5<br>D5 ← P5 P6 Q5 Q6<br>D6 ← P6 P1 Q6 Q1 | D1 ← P1 P2 Q1 Q2 R1 R2<br>D2 ← P2 P3 Q2 Q3 R2 R3<br>D3 ← P3 P4 Q3 Q4 R3 R4<br>D4 ← P4 P5 Q4 Q5 R4 R5<br>D5 ← P5 P6 Q5 Q6 R5 R6<br>D6 ← P6 P1 Q6 Q1 R6 R1 |
| $D_P = 4$ | D1 ← P1 P2<br>D2 ← P2 P3<br>D3 ← P3 P4<br>D4 ← P4 P5<br>D5 ← P5 P6<br>D6 ← P6 P1<br>E1 ← P1 P2<br>E2 ← P2 P3<br>E3 ← P3 P4<br>E4 ← P4 P5<br>E5 ← P5 P6<br>E6 ← P6 P1 | D1 ← P1 P2 Q1 Q2<br>D2 ← P2 P3 Q2 Q3<br>D3 ← P3 P4 Q3 Q4<br>D4 ← P4 P5 Q4 Q5<br>D5 ← P5 P6 Q5 Q6<br>D6 ← P6 P1 Q6 Q1<br>E1 ← P1 P2 Q1 Q2<br>E2 ← P2 P3 Q2 Q3<br>E3 ← P3 P4 Q3 Q4<br>E4 ← P4 P5 Q4 Q5<br>E5 ← P5 P6 Q5 Q6<br>E6 ← P6 P1 Q6 Q1 | D1 ← P1 P2 Q1 Q2 R1 R2<br>D2 ← P2 P3 Q2 Q3 R2 R3<br>D3 ← P3 P4 Q3 Q4 R3 R4<br>D4 ← P4 P5 Q4 Q5 R4 R5<br>D5 ← P5 P6 Q5 Q6 R5 R6<br>D6 ← P6 P1 Q6 Q1 R6 R1<br>E1 ← P1 P2 Q1 Q2 R1 R2<br>E2 ← P2 P3 Q2 Q3 R2 R3<br>E3 ← P3 P4 Q3 Q4 R3 R4<br>E4 ← P4 P5 Q4 Q5 R4 R5<br>E5 ← P5 P6 Q5 Q6 R5 R6<br>E6 ← P6 P1 Q6 Q1 R6 R1 |

(left) shows the results of this experiment, which are consistent with the predictions. The CPU time required to select a rule instance generally seems unaffected by the number of definitions in long-term memory. There is a slight upturn at the curve's end, but this falls within the 95 percent confidence interval. Experiments with larger knowledge bases would clarify the situation further, but the evidence to date is that this aspect of the analysis is correct.

A second hypothesis relates the processing time needed to select a rule to the 'branching factor' of definitions from the predicates that appear in their antecedents:

- *Processing time per rule selection $T_D$ is a linear function of the average number of definitions per predicate $D_P$.*

To check this prediction we varied the number of definitions per predicate ($D_P$ = 2, 4, 6, 8, 10) while holding the beliefs per predicate constant at $B_P$ = 3. We repeated this at four settings for the number of antecedents per definition. Figure 2 (right) presents the results with separate curves for $A_D$ = 2, 4, 6, and 8, the first three corresponding to columns in Table 2. These generally support the hypothesis, as each of the four curves are approximately linear in $D_P$. The lowest curve looks flat, but inspection of the numbers reveals that it increases as well, although slowly.



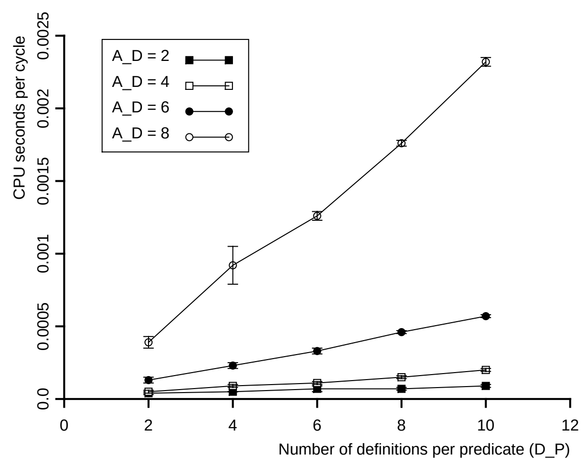




*Figure 1.* The processing time per inference cycle in CPU seconds as a function of (left) the total number of definitions ($N_D$) and (right) the number of definitions per predicate ($D_P$). For the first plot, we set $A_D = 3$, $D_P = 3$, and $B_P = 3$; for the second graph, we set the factor $B_P = 3$.

A third implication of the formal analysis is that the time needed to select an instance of a definitional rule grows exponentially with the average rule complexity:

- *Processing time per rule selection $T_D$ is an exponential function of the average number of antecedents per definition $A_D$.*

We tested this claim by varying the number of antecedents per definition ($A_D$ = 2, 4, 6, 8) while holding the beliefs per predicate constant at $B_P$ = 3. We examined PENUMBRA's behavior at different settings for the number of definitions per predicate. Figure 2 (left) displays the CPU time per inference cycle, giving separate curves for $D_P$ = 2, 4, 6, 8, and 10, which we obtained by extending the rule patterns in Table 2. These differ in the number of definitions, which we could not hold constant in this study. All five curves are nonlinear, which is consistent with the prediction of exponential growth in $A_D$, the antecedents per definition. This is not especially concerning, as PENUMBRA's definitions should typically have fewer than eight antecedents.

A final hypothesis states that the computation time needed for rule selection is polynomial in the number of beliefs in working memory that involve the same predicate as the focus belief:

- *Processing time per rule cycle $T_D$ is a polynomial function of the average number of beliefs per predicate $B_P$.*

To evaluate this conjecture, we varied the beliefs per predicate ($B_P$ = 1, 2, 3, 4, 5) while holding the other factors constant ($A_D$ = 6, $D_P$ = 3, $N_D$ = 6). As Figure 2 (right) reveals, the increase in CPU time per inference cycle with $B_P$ seems approximately linear, which is better than the polynomial growth predicted by the analysis. However, we should extend the curve with higher settings to increase confidence in this conclusion. This relationship is a greater concern than exponential growth in $A_D$, as the number of beliefs per predicate can increase over an extended run.

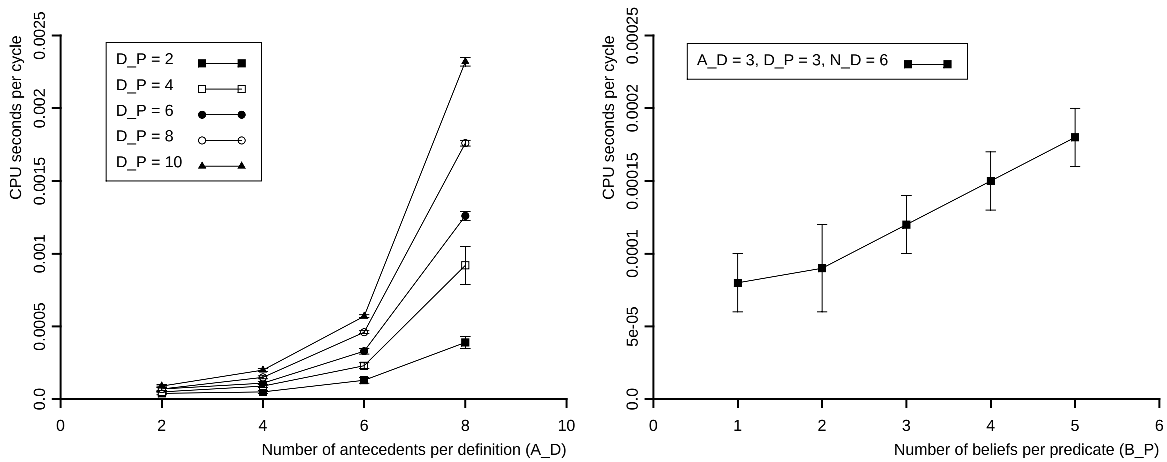



*Figure 2.* The processing time per inference cycle in CPU seconds as a function of (left) the number of antecedents per definition and definitions per predicate and (right) the number of beliefs per predicate. For the first plot, we set the factor $B_P = 3$; for the second graph, we set $N_D = 6$, $A_D = 3$, and $D_P = 3$.

In summary, the results from our experimental studies of definition selection were generally consistent with the predictions that follow from our formal analysis. They indicate that processing time is either constant or grows slowly with the number of definitions, the number of definitions per predicate, and the number of beliefs per predicate. As expected, the only factor that causes exponential growth is the number of antecedents per definition, which results from the need to consider partial matches in support of abductive inference. However, we can bound this cost by limiting the number of antecedents associated with each rule, which suggests an important role for the hierarchical organization of such knowledge.

## 4. Scalability of Explanation Construction

Although efficient behavior at the level of the inference cycle is important, it is not enough on its own. The process of generating full explanations must also operate in a scalable manner. For example, we would like the time needed to construct a good account to increase slowly with the number of observations and with the complexity of the resulting structures. Informal analysis suggests that the number of inference cycles should grow as a linear function of both factors, but we will not address them here. Instead, we will focus on an even more important matter – scalability to the number of alternative explanations – that arises because the task involves combinatorial search.

Consider a phenomenon well known in human language processing. Some sentences have many possible parses, yet we take little if any more time to understand them than ones with fewer parses. Given what we know about human cognition, the natural conclusion is that this occurs because people draw on effective heuristics to guide their choices. Such heuristics can occasionally mislead processing, as demonstrated by the existence of garden path sentences (e.g., *The old man the boats*), but these are exceptions that prove the rule of thumb. PENUMBRA relies on heuristics at two key choice points: selection of focus beliefs and selection of definitions. This suggests another hypothesis about the system's behavior:

- *Given effective heuristics, the time needed to find the best explanation is independent of the number of consistent worlds.*

In other words, during its search through the space of explanations, PENUMBRA should find the best candidate first and should not be distracted by competing accounts that have lower quality. However, before we can test this claim, we must first make it operational. This means that we must specify what we mean by the term 'best' and which heuristics are likely to be effective.

### 4.1 Explanation Quality and Heuristics

The notion that one explanation is 'best' implies some criterion for determining their quality.[1] A classic criterion is *simplicity*, which favors accounts that involve fewer reasoning steps or that rely on fewer assumptions (e.g., Peng & Reggia, 1986). However, Ng and Mooney (1990) have argued instead for using *coherence*, which they defined in terms of the number of ways that beliefs are linked to each other through available knowledge. Another approach associates numbers with individual rules, possible assumptions, or both. For instance, Hobbs et al.'s (1993) abduction system assigned weights to each possible assumption and ranked alternative explanations by the summed weights of their assumptions, with lower totals being better, while Appelt and Pollack (1992) took a similar approach. Parsers that rely on probabilistic context-free grammars instead assign probabilities to each rule, then compute a posterior probability for candidate parse trees by multiplying the scores for each rule involved in them (e.g., Charniak & Shimony, 1990; Gordon, 2018).

We will not take a position here about which of these criteria is more desirable, but, for the sake of illustration, we will adopt a variant of Hobbs et al.'s scheme in testing our hypothesis. Suppose that each PENUMBRA definition comes with a score between zero and one, and that the quality of an explanation is the average score of its component rule instances. This suggests that we use a criterion that favors selection of candidate rules with higher scores. This is similar in spirit to conflict resolution strategies in production system architectures that prefer rules with higher strengths (e.g., Anderson & Lebiere, 1998). Given that PENUMBRA can apply rules with unsatisfied antecedents, we might modulate this score, say multiplying it by the fraction of matched conditions. This will not be relevant to the experiments reported later, which involve purely deductive reasoning, but it could play a role in our future work.

However, an effective heuristic for rule selection is not enough for PENUMBRA to generate high-quality accounts earlier than it considers low-quality ones. For this ability, the system must also carry out depth-first search through the space of explanations by applying definitional rules that elaborate on the most promising world before turning to others. This strategy will sometimes lead it to extend worlds that were not the best choice in hindsight, as occurs with garden path sentences, but that is the nature of heuristics. Nevertheless, they often recommend the right option, in this context leading the elaborative inference to construct the highest-quality explanation before the search process begins to consider alternatives.

PENUMBRA's criterion for selecting focus beliefs is the mediator that determines whether it behaves in a depth-first manner. If the system selects a focus at random, then it will tend to switch

1. Some authors (e.g., Eckroth & Josephson, 2014) have defined abduction as 'inference to the best explanation', but not all explanations need be abductive and finding the best should not be part of the task's definition.

*Table 3.* A simple subset of English syntax stated as a context-free grammar, each with an associated score, and five parses of the sentence *A dog chased the cat with a stick on the roof* with this grammar.

| *Grammatical Rules* | *Score* | *Allowed Parses* | *Score* |
|---|---|---|---|
| *S → NP VP* | *1.0* | *((A dog) (chased (the cat (with (a stick))) (on (the roof))))* | *0.726* |
| *VP → V NP* | *0.4* | *((A dog) (chased (the cat) (with (a stick)) (on (the roof))))* | *0.716* |
| *VP → V NP PP* | *0.9* | *((A dog) (chased (the cat (with (a stick)) (on (the roof)))))* | *0.721* |
| *VP → V NP PP PP* | *0.5* | *((A dog) (chased (the cat) (with (a stick (on (the roof)))))))* | *0.721* |
| *NP → ART N* | *0.5* | *((A dog) (chased (the cat (with (a stick (on (the roof))))))))* | *0.679* |
| *NP → ART N PP* | *0.2* | | |
| *NP → ART N PP PP* | *0.8* | | |
| *PP → P NP* | *0.9* | | |

between elaborating one world and extending others, approximating breadth-first search. In contrast, if it prefers to focus attention on more recently generated beliefs, then it will elaborate on the current world before it extends others.[2] Such a bias has been a common method for conflict resolution in production system architectures, and Young (1982) has noted that it offers a natural way to model depth-first strategies. To keep this scheme from producing infinite loops, it is often combined with a 'refraction' technique that forbids reapplying the same rule instance until its matched elements are refreshed. PENUMBRA mimics this mechanism by decreasing a belief's score once it has served as the focus, redirecting attention to unused elements instead.

Recall that applying a definitional rule, even in the purely deductive case, produces at least one new belief: its instantiated head. If the system focuses on more recent elements, then it will first consider rules that match it and extend the current explanation. An example should clarify this effect. Table 3 presents a context-free grammar for a subset of English that includes prepositional phrases, which are an important source of ambiguity. PENUMBRA encodes this syntactic knowledge differently in that it includes explicit constraints (e.g., the same prepositional phrase cannot be attached to both a noun and a verb), but the traditional notation will simplify our discussion. Also, each definition has an associated weight that indicates its preference relative to others. Suppose we provide PENUMBRA with this knowledge and with the sentence *A dog chased the cat with a stick on the roof*. Table 3 shows five parses supported by the grammar, along with the average score of rules used to construct them and the resulting scores for each interpretation.

For the sake of argument, assume that we present PENUMBRA with the entire sentence at once, rather than incrementally. On each inference cycle, the system focuses on the most recently created belief that has not led to a rule application. Combined with a heuristic for selecting the retrieved (through connection with the focus) and matched definition with the highest score, this bias should generate the first parse tree in the table before it considers the others. PENUMBRA should first produce beliefs for the four noun phrases and then embed the latter two in prepositional phrases. Next, the system would decide to include *(with (a stick))* in *(the cat (with (a stick)))*, after which it would include this noun phrase and the prepositional phrase *(on (the roof))* into the verb phase

2. Because PENUMBRA represents worlds in a distributed fashion, each inference can extend more than one explanation.

*(chased (the cat (with (a stick))) (on (the roof))*. Finally, it would combine this constituent with the noun phrase *(a dog)* to produce the first parse tree in the table.

Of course, PENUMBRA would not encode this parse tree as a list structure, but rather as as a set of beliefs, including nonterminal predicates, that are connected through rule instances. Moreover, the system can reuse many of these elements (especially those at lower levels) in other parses that it would consider later, since they would share many constituents. Each alternative parse would be associated with a distinct world, but these would be stored as markers on working memory elements, so that common beliefs need only be inferred once. However, we are concerned here not with how PENUMBRA can generate all parses, but with how it can arrive at the best-scoring explanation before even considering these other sentence interpretations.

### 4.2 Experiments on Explanation Scalability

We can build on this informal analysis to to test our hypothesis about the scalability of explanation construction in PENUMBRA. This predicts that, as we increase the number of different accounts that are internally consistent, the time taken to find the best candidate will be unaffected. Whether the two heuristics we discussed earlier – recency for the selection of focus beliefs and definition score for the selection of rules – are effective enough to produce this result is an empirical question, but the parsing example gives reasonable cause for hope. Because we have already examined the scaling of computational costs per inference cycle, we will use here number of inference steps as the dependent variable rather than CPU time.

Again, the main independent factor is the number of consistent explanations for a given set of observations that are supported by PENUMBRA's knowledge base. However, we also want to hold constant other possible influences on processing time. We have already addressed factors like the number of rules, their number of antecedents, and the number of beliefs per predicate in our study of cycle time. The two remaining influences are the number of observations to be explained and the complexity of explanations. The parsing task is a convenient setting to test the hypothesis because it is familiar, it avoids complications raised by the need to introduce default assumptions during abduction, and it lets us easily control the number of observations (sentence length). Moreover, different parse trees require approximately the same number of inferences to construct them.

However, it would seem difficult to vary the number of explanations without also altering these two factors. Fortunately, we can use a simple strategy to vary the number of alternative parses while holding the others constant. Briefly, we can start with a general grammar like the one in Table 3 and restrict it by removing rules or by splitting nonterminal symbols. For example, deleting the third *VP* rule will eliminate one of the five parses. Similarly, replacing *PP* with *PP1* in some cases and *PP2* in others, along with replacing $PP \rightarrow P\ NP$ by $PP1 \rightarrow P1\ NP$ and $PP2 \rightarrow P2\ NP$, allows only two parses. This change also requires modifying rules that associate parts of speech with particular words like *with* and *on*, which we have omitted from the table.[3] We used this approach create three variations on the initial grammar that accepted different subsets of the original five parses of the sentence *A dog chased the cat with a stick on the roof*, which we held constant across the study.

3. The approach does require altering the number of definitional rules, and possibly the number of constraints, but we are not concerned with time per cycle here, only the number of inferences needed to find the best explanation.

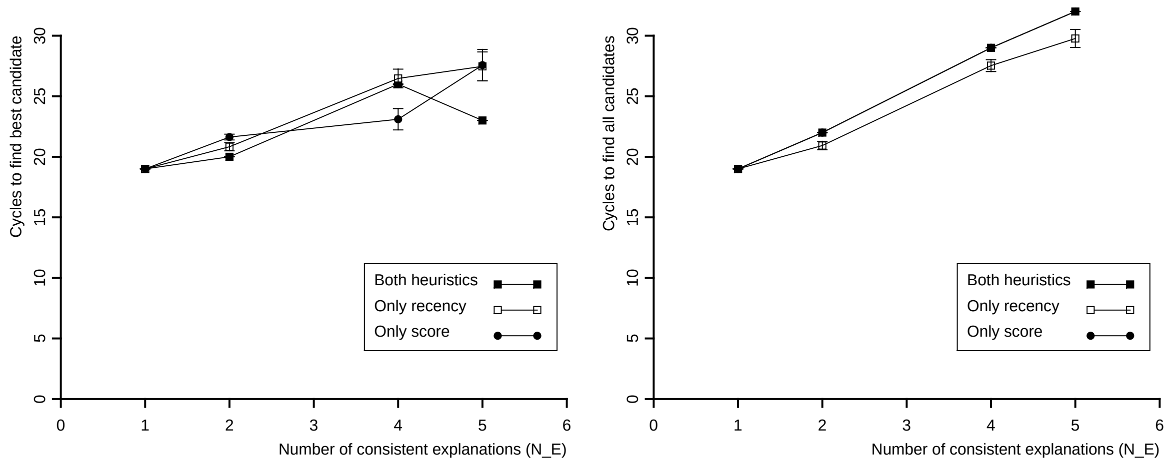



*Figure 3.* Number of cycles (left) to find the best parse with variants of the grammar in Table 3 and (right) to find all consistent parses using as heuristics both belief recencies and rule scores, only rule scores, and only belief recencies to guide the search process.

We should also note that our core hypothesis comes with a corollary. Because the ability to find the best explanation before others depends on use of effective heuristics, this should not occur when PENUMBRA instead selects focus beliefs or definitions at random. The earlier informal analysis suggests that both are prerequisites for finding the best account earlier than its competitors. Thus, our experiment should vary two factors: the number of consistent explanations (in this case the number of parses) and whether the system relies on the plausible heuristics described earlier to select focus beliefs and rule instances. We should note that other measures of explanation quality, say ones that revolve around coherence, are definitely possible, but we will postpone examining their empirical behavior until future research.

Because PENUMBRA stores beliefs in a distributed manner, by marking them with worlds in which they do not hold, it is not obvious at first glance how to determine when it has found the best explanation. However, the system stores with each world the inference cycle on which it was modified most recently. If we run it until completion, that is, until it makes no more inferences and thus has found all consistent worlds for the given observations, then we can find both the best-scoring world and the cycle on which it was last elaborated. We can use this number as the dependent measure of when the system found the best explanation. Of course, running PENUMBRA until it exhaustively finds all consistent explanations goes against the purpose of heuristic guidance, but we are relying on it only to test our hypothesis. If that claim holds, then in future studies it can halt after finding the first few explanations, since we will know they are high quality.

We followed this strategy using four variants of the grammar in Table 3, running the system 30 times and averaging the results. Figure 3 (left) presents the findings for this experiment. The $x$ axis denotes the number of parses that are possible for the observed word sequence and the $y$ axis encodes the inference cycle on which PENUMBRA found the best-scoring parse. The graph shows three separate curves. One line represents the system's behavior when it uses a recency heuristic to select focus beliefs and rule scores to select which rule instance to apply. Contrary to predictions, the number of inference cycles needed to find the best parse grows slowly with the number of alternatives, although there is an encouraging dip on the final point. Inspection of system traces revealed that it encountered constraint violations midway through the runs, indicating that it was not pursuing the full depth-first search as intended. Another line shows PENUMBRA's behavior when it instead selected focus beliefs at random and the third line shows when it chose rule instances at random. In both conditions, the number of inferences needed to find the best parse increases at about the same rate as when using both criteria. This suggests that the two heuristics are doing no better than chance, so developing more effective ones should have high priority.

We also recorded the number of inference steps needed to find all parses for the sentence. The heuristics should play no role here, as the system must generate all possible accounts, regardless of their order. Figure 3 (right) shows that these curves fall only slightly above that for the cycles needed to find the best interpretation. The reason is that, when multiple parses were possible, they shared most of their constituents, which meant that only a few inferences remained to generate alternative parses after PENUMBRA found the first one. Thus, although the two heuristics provided less assistance than predicted for finding the best explanation before other candidates, this ability may be less important than expected in many practical settings.

## 5. Related Research

The research we have reported here draws on ideas from a variety of traditions in artificial intelligence. We have already mentioned some prior work on explanatory inference, but we have been influenced the most by PENUMBRA's direct predecessors – AbRA (Bridewell & Langley, 2011) and UMBRA (Meadows et al., 2014) – that share its alternation between selecting a focus belief and deciding which rule to apply. As discussed elsewhere (Langley & Meadows, 2019), the main differences revolve around the use of constraints to handle inconsistencies and related support for multiple worlds. Our concern with scalability has its origins in work on efficient matching in production systems, which led to algorithms like Rete (Forgy, 1982) and TREAT (Miranker, 1987). However, these assumed all-or-none matching, which does not suffice for abduction scenarios that involve partial satisfaction of rules' antecedents. PENUMBRA's use of focus beliefs is a direct response to this issue, and our analysis of inference cycle costs took it into account, even though our experiments addressed purely deductive tasks.

Within the literature on cognitive architectures, the closest relative is ACT-R (Anderson & Lebiere, 1998), which provides a small set of buffers, each of which can hold a single element. On each cycle, the content of a buffer leads to the retrieval of some production rule, which on application produces new short-term structures. Thus, the buffers act as bottlenecks through which information must pass, effectively focusing cognitive attention on their elements. The motivations behind this buffer mechanism are different from the ones that led to PENUMBRA's analog, and our framework allows only one focus belief per cycle, but at an abstract level the similarities are striking. We are not aware of any analyses that characterize the computational cost of ACT-R's processing, but many of the same factors should be relevant. In contrast, the Soar architecture (Laird, 2012) ensures efficient processing on each cycle by constraining the organization of working memory and the conditions of rules that match against its elements.

At a higher level, research on abductive reasoning has dealt extensively with search through the space of explanations. Some approaches, including answer set programming (e.g., Baral, 2003) and probabilistic parsing (e.g., Charniak & Shimony, 1990), have relied on exhaustive generation of alternatives, ranking candidates only afterwards. Other techniques have combined numeric heuristics with more selective methods, from beam search (e.g., Gordon, 2018; Ng & Mooney, 1990) to branch and bound (Hobbs et al., 1993). However, these have lacked PENUMBRA's alternation between focus and rule selection, which offers greater opportunities for search guidance, but more chances to be led astray. As noted earlier, the distributed representation of competing worlds also supports a form of implicit parallelism that reduces reliance on error-free heuristics. This idea bears a close resemblance to the approach used in assumption-based truth maintenance systems (de Kleer, 1986, 1994). These adopt a similar encoding to consider multiple interpretations of observations during an abduction-like process and some variants scale very well to large problems.

## 6. Closing Remarks

In this paper, we reviewed a computational account of explanatory inference that distinguishes between two forms of long-term knowledge – definitions and constraints – and between three types of dynamic beliefs – observed, abduced, and derived. The theory further posits that beliefs are organized into linked justifications and associated with one or more worlds in which they hold. The framework includes roles for three primary mechanisms: focusing attention on an existing belief, applying definitions to generate new beliefs, and repairing violated constraints when they are detected. We also reviewed PENUMBRA, an implemented system that incorporates these theoretical assumptions about the representation and processing of explanatory structures.

After this, we examined how the system scales to complexity. We presented a formal analysis of the computational costs that arise within PENUMBRA's three processing stages and reported empirical studies of the most expensive module, which selects a definition to apply. Results from controlled experiments with a synthetic knowledge base agreed with predictions from the analysis, show that processing time scales well on every factor except antecedents per definition, which we can bound in practice. We also reported tested qualitative hypotheses about the system's ability to search through the space of explanations, in particular that it would find the best account before considering alternatives. However, PENUMBRA's behavior on an ambiguous parsing task showed its heuristics for selecting foci and definitions to apply were less effective than we had expected.

In summary, the empirical studies to date support some of our hypotheses about PENUMBRA's scalability, they have called others into question. Future research should analyze in greater detail the reasons why the system has difficulty finding good accounts before poor ones, and we should attempt to replicate the positive findings with more complex reasoning problems. Our next experiments should include explanation tasks that require the introduction of plausible assumptions and they should examine scaling issues for selecting focus beliefs and constraint processing to complement our initial results on choosing definitions. Moreover, we should test the system's abilities on large knowledge bases that some researchers have developed, such as the Monroe corpus (Blaylock & Allen, 2005), and extended behavior streams that others have collected, such as the Triangle-COPA problem set (Maslan, Roemmele, & Gordon, 2015).

In addition, we should explore other criteria for explanation quality and associated heuristics. One approach would calculate probabilities for alternative accounts and use them to guide search through the space of candidates. Another would adopt Ng and Mooney's (1990) notion of explanatory coherence, which reflects people's preference for narratives that are tightly interconnected. We should also examine other heuristics for selecting the focus beliefs that serve as mediators for rule application. One promising idea involves favoring candidates that hold in approximately half of the active worlds, as rules that chain off them may provide more information. The PENUMBRA architecture, with its user-modifiable parameters, offers a promising computational framework for further exploration of strategies for scalable explanatory inference.

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