Scales and Hedges in a Logic with Analogous Semantics

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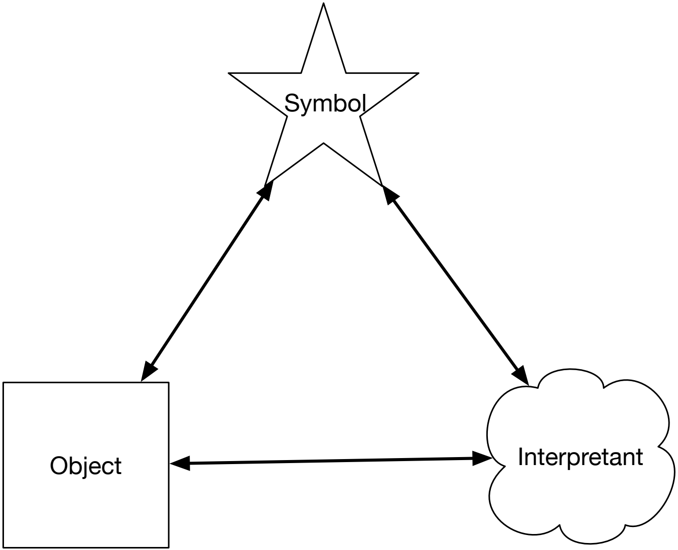
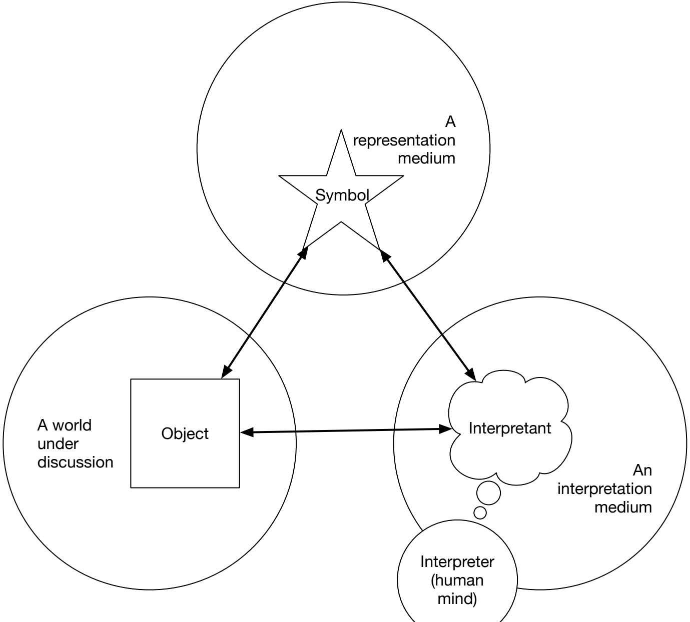
Semiotics



following Pierce

Different Worlds



Empathy and Meaning in Ethical Decision Making

- The ability to imagine may be critical for pro-social behaviours.
- Current literature distinguishes
  - affective empathy: ability to share another's feelings and emotions, compassion and distress (Hodges & Myers, 2007).
  - cognitive empathy: inferences about anothers' mental states, ability to perceive their intentions, motivations and expectations.

⇒ Evidence from pathologies that present with impairment in empathy: psychopathy and autism.

  - Psychopaths preserve cognitive empathy but have a low affective empathy (Smith, 2006) ⇒ anti-social behaviour, insensibility towards signs of distress
  - People with autism show deficits in cognitive empathy but have their affective empathy intact (Smith, 2006) ⇒ normal morality

⇒ Decision making needs imagination of what consequences mean

- E.g.: invest in climate change action or economic growth?

Demo

https://logical-lateration.appspot.com/video.html

Alternatives to Set-Theoretical Semantics

- Modern set-theoretical mathematical logic originated in the 19th century (before that no formal logic!)
- 20th century:
  - Alternatives to set-theory: Leśniewski's Protothetic, a mereological foundation for logic
    - Mereology: "flat" foundational relation *part-of* between individuals at the same level instead of hierarchical *element-of*

    ⇒ Influential followers (e.g., Tarski)

    ⇒ Advantageous, in particular, for continuous domains, e.g.: time/space

    ⇒ In terms of mathematical structures: mereology describes a lattice or semi-lattice structure, which is a generalization of Boolean Algebra
  - Many equivalences between logics and mathematical theories, in general
  - Logically, close relationship between, e.g.: intuitionistic logic, certain modal logics, description logics, and Fuzzy Logic (Hájek, 1998) as based on lattices or semi-lattices
  - As computational structures: directed acyclic graphs (including trees) also belong into this same category
  - Generally, any partial order gives rise to a lattice structure

⇒ Context Logic a cognitively motivated logic with a lattice semantics

A Reasoner with Analogous Semantics

- Analogous semantics of Fuzzy Logic (Zadeh): Pixel region *shoe*123 has RGB color value with R-component of 90% "the shoe is red to the degree 90%"
  - ⇒ Color of the shoe can be reconstructed from statement and degree
  - ⇒ Truth and meaning in the intuitive sense of correspondence with reality
  - ⇒ Fuzzy Control Systems: can interact with the world, be verified logically
  - ⇒ But: meaning of a text is still just one numerical truth value
  - ⇒ Context Logic (CL)
- Vector Symbolic Architectures (Kanerva): abstract distributed model of cognition
  - Intensional pointer/association semantics
  - Advantage: input image and all symbols are of the same type (vectors)
  - But: no analogous semantics

⇒ Activation Bit Vector Machine: a VSA for logical reasoning with full analogous semantics for complex CL formulae

Context Logic: a Two-layered Logic for Reasoning about, and in Context

1. *Context terms* $\mathcal{T}_C$ are defined over a set of variables $\mathcal{V}_C$:
   - Any context variable $v \in \mathcal{V}_C$ and the special symbols $\top$ and $\bot$ are atomic context terms.
   - If c is a context term, then its complement $(\sim c)$ is a context term.
   - If c and d are context terms then the intersection $(c \sqcap d)$ and sum $(c \sqcup d)$ are context terms.
2. *Context formulae* $\mathcal{F}_C$ are defined as follows:
   - If c and d are context terms then $[c \sqsubseteq d]$ (c is *subcontext* of d) is an atomic context formula.
   - If ϕ is a context formula, then $(\neg\phi)$ is a context formula.
   - If ϕ and ψ are context formulae then $(\phi \wedge \psi)$, $(\phi \vee \psi)$, $(\phi \rightarrow \psi)$, and $(\phi \leftrightarrow \psi)$ are context formulae.
   - If $x \in \mathcal{V}_C$ is a variable and ϕ is a formula, then $\forall x : \phi$ and $\exists x : \phi$ are context formulae.

Context Logic: Conventional Semantics

- Syntax gives rise to a hierarchy of languages
  - CLA fragment (atomic CL) allows only $\wedge$
  - CL0 (propositional CL) allows any construction without quantifiers
  - CL1 (first order CL) adds quantifiers
- Conventional semantics: several different characterizations
  - Set-theoretical semantics (2007)
  - Kripke semantics (2008)
  - FOL axiomatic characterization based on Partial Orders and using DAGs (2008, 2009, 2012, and here)
  - Category-theoretical semantics (2021)

Context Logic: Semantics by Axiomatic Characterization I

- The core relation $\sqsubseteq$ is a partial order, i.e. (2012):
  - reflexive

$$[\alpha \sqsubseteq \alpha]$$

  - antisymmetric

$$[\alpha \sqsubseteq \beta] \wedge [\beta \sqsubseteq \alpha] \rightarrow \forall \xi : ([\alpha \sqsubseteq \xi] \rightarrow [\beta \sqsubseteq \xi]) \wedge ([\xi \sqsubseteq \alpha] \rightarrow [\xi \sqsubseteq \beta])$$

  - transitive

$$[\alpha \sqsubseteq \beta] \wedge [\beta \sqsubseteq \gamma] \rightarrow [\alpha \sqsubseteq \gamma]$$

- Characterization of $\sqsubseteq$ in relation to $\rightarrow$:

$$[\beta \sqsubseteq \gamma] \rightarrow \forall x : [x \sqsubseteq \beta] \rightarrow [x \sqsubseteq \gamma]$$

$\Rightarrow$ Allows us to move any complex context term to the right hand side: can characterize the relation between the context term operators and the logical operators with respect to their occurrence on the right hand side.

Context Logic: Semantics by Axiomatic Characterization II

- Intersection ($\sqcap$) and conjunction ($\wedge$):

$$[\top \sqsubseteq \alpha \sqcap \beta] \leftrightarrow [\top \sqsubseteq \alpha] \wedge [\top \sqsubseteq \beta]$$

- Sum ($\sqcup$) and disjunction ($\vee$):

$$[\top \sqsubseteq \alpha \sqcup \beta] \leftrightarrow \forall \xi : \exists \chi : [\chi \sqsubseteq \xi] \wedge ([\chi \sqsubseteq \alpha] \vee [\chi \sqsubseteq \beta])$$

- Complement as related to $\bot$ and (with empty domains excluded, i.e., $\neg[\top \sqsubseteq \bot]$) to $\neg$:

$$[\top \sqsubseteq {\sim}\alpha] \leftrightarrow [\alpha \sqsubseteq \bot]$$

- Can immediately derive relations:
  - Converse: $[\alpha \sqsupseteq \beta] \overset{def}{\Leftrightarrow} [\beta \sqsubseteq \alpha]$
    Useful to specify opposites (north/south, small/tall, etc., 2020)
  - Equality: $[\alpha = \beta] \overset{def}{\Leftrightarrow} [\alpha \sqsubseteq \beta] \wedge [\beta \sqsubseteq \alpha]$
  - Overlap: $[\alpha \bigcirc \beta] \overset{def}{\Leftrightarrow} \neg[\alpha \sqcap \beta \sqsubseteq \bot]$
  - Non-empty subcontext: $[\alpha \dot{\sqsubseteq} \beta] \overset{def}{\Leftrightarrow} [\alpha \bigcirc \beta] \wedge [\alpha \sqsubseteq \beta]$

Context Logic: Semantics by Axiomatic Characterization III

- Using the conjunctive term operator $\sqcap$, arbitrary further p.o. relations can be constructed (basic property of lattice structures) without introducing special relational symbols
- Allows us to understand certain contexts as relational or dimensional contexts

$\Rightarrow$ Will allow us to analyze any relation in terms of one singular relation $\sqsubseteq$

1. Partial orders
2. General (non-partial-order) relations in terms of a mereological description of graphs/tuples

- Wide range of relations are partial orders, e.g.: *spatial-part-of* or *north-of*.
- Transitivity of p.o.s is fundamental for reasoning in many domains.
  - reflexivity $[\alpha \sqcap x \sqsubseteq \alpha]$
  - antisymmetry $[\alpha \sqcap x \sqsubseteq \beta] \wedge [\beta \sqcap x \sqsubseteq \alpha] \rightarrow [\beta \sqcap x = \alpha \sqcap x]$
  - transitivity $[\alpha \sqcap x \sqsubseteq \beta] \wedge [\beta \sqcap x \sqsubseteq \gamma] \rightarrow [\alpha \sqcap x \sqsubseteq \gamma]$

Context Logic: Semantics by Axiomatic Characterization IV

The proofs follow via:

$$[\alpha \sqcap x \sqsubseteq \beta] \leftrightarrow [\alpha \sqcap x \sqsubseteq \beta \sqcap x]$$

because $\alpha \sqcap x \sqsubseteq x$ is trivially true.

- We can define a *contextualization* syntax as a shorthand:

$$x : [\alpha \sqsubseteq \beta] \overset{def}{\Leftrightarrow} [\alpha \sqcap x \sqsubseteq \beta].$$

- This already brings us close to the conventional way, we write relations in FOL
- We can say, that this conventional syntax is a further abbreviation, a schema, or syntactic sugar: $x[\alpha, \beta] \overset{def}{\Leftrightarrow} [\alpha \sqcap x \sqsubseteq \beta]$.
- This statement is in CLA, i.e., on the most fundamental expressive level

⇒ We have the reasoning back-bone of any partial order “for free”, i.e., without requiring any axiomatization

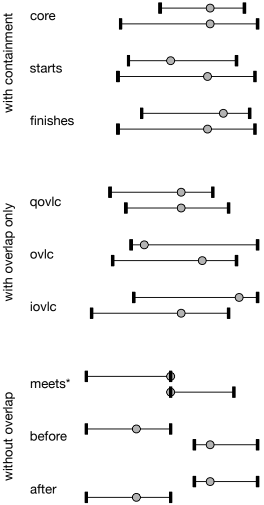
Basic Vocabulary and Semantic Classes with Dimensional Meanings

Type	Dimension	Comp.
Structural	-	is
Structural	-	are
PP (static)	north-south (+)	north
PP (static)	north-south (-)	south
PP (static)	east-west (+)	east
PP (static)	east-west (-)	west
PP (static)	size (+)	large
PP (static)	size (-)	small
PP (static)	left-right (-)	left
PP (static)	left-right (+)	right
PP (static)	left-right (*)	side (adv.)
PP (dynamic)	up-down (+)	up
PP (dynamic)	up-down (-)	down
PP (dynamic)	to-from (+)	to
PP (dynamic)	to-from (-)	from
Aspect	contains-during (+)	V-ing
Tense	before-after (+)	will V

Type	Dimension	Comp.
Verb (i)	spatial (obj-ext.)	move
Verb (i)	spatial (obj-ext.)	run
Verb (i)	spatial (obj-ext.)	go
Verb (i)	spatial (obj-ext.)	continue
Verb (i)	health (+, neg to avg)	recover
Verb (t)	spatial (obj-ext.)	drive
Verb (t)	spatial (subj-arm)	pull
Verb (t)	health (+, neg to avg)	heal
Verb (t)	health (-)	harm
Verb (t)	health (min)	kill
Verb (t)	poss. space (subj, -)	give
Verb (t)	poss. space (subj, +)	receive
NP	-	a/the trolley
NP	-	a/the track
NP	-	an/the agent
NP	-	a/the lever
NP	-	one person
NP	-	five people

Temporal Relations

relation name	causation (c)	containment (t)
$core(i, j)$	$i \sqcap c = j \sqcap c$	
$starts(i, j)$	$i \sqcap c \sqsubset j$	$i \sqcap t \sqsubset j$
$finishes(i, j)$	$j \sqcap c \sqsubset i$	
$icore(i, j)$	$i \sqcap c = j \sqcap c$	
$istarts(i, j)$	$i \sqcap c \sqsubset j$	$j \sqcap t \sqsubset i$
$ifinishes(i, j)$	$j \sqcap c \sqsubset i$	
$qcore(i, j)$	$i \sqcap c = j \sqcap c$	
$qstarts(i, j)$	$i \sqcap c \sqsubset j$	$j \sqcap t = i \sqcap t$
$qfinishes(i, j)$	$j \sqcap c \sqsubset i$	
$qovlc(i, j)$	$i \sqcap c = j \sqcap c$	
$ovlc(i, j)$	$i \sqcap c \sqsubset j$	$i \sqcap t \circ j$
$iovlc(i, j)$	$j \sqcap c \sqsubset i$	
$meets(i, j)$	$i \sqcap c = j \sqcap c$	
$before(i, j)$	$i \sqcap c \sqsubset j$	$\neg[i \sqcap t \bigcirc j]$
$after(i, j)$	$j \sqcap c \sqsubset i$	



General Relations

- But we need the expressive power of CL0 and CL1 for general relations
- With the existential quantitfier $\exists$, CL1 allows any other relation to be constructed including non-p.o. relations, such as the instance-of relation between an object *o* and a class *c*:

$$\exists e : [e \sqsubseteq isi] \wedge [o \sqcap e \dot{\sqsubseteq} a_1] \wedge [c \sqcap e \dot{\sqsubseteq} a_2].$$

  o is an *instance of c* iff there is a subcontext *e* of *isi* (intuitively, the edge *e* of the graph *isi*) of so that *o* overlaps *e* in a_1 (first arguments or ends of edges) and *c* overlaps *e* in a_2 (second arguments or tips of edges).
- Formally, this construction is called a *tuple generator*, with which arbitrary relations can be constructed.
- We can then again say that the more conventional relation statement $isi(\alpha, \beta)$ is only a schema, an abbreviation, syntactic sugar

$$isi(\alpha, \beta) \overset{def}{\Leftrightarrow} \exists e : [e \sqsubseteq isi] \wedge [\alpha \sqcap e \dot{\sqsubseteq} a_1] \wedge [\beta \sqcap e \dot{\sqsubseteq} a_2].$$

Verbs

- Action verbs (CL1, 2021) generate event parameters (Davidson) that tie together the parts of a sentence (tuple generator construction, hypergraph edge)
- Static predication sentences

$$\langle subj\rangle(is|are)\langle pred^{\sigma}\rangle\langle obj\rangle \mapsto \langle dim\rangle^{\sigma}[\langle subj\rangle, \langle obj\rangle]$$

- Action verb sentences
- Intransitive verb with adverbial PP, e.g.: *Toms goes down the street*

$$\langle subj\rangle\langle verb_i^{\sigma_v}(a,t)\rangle\langle pred^{\sigma_p}\rangle\langle obj^p\rangle \mapsto$$
$$\exists e_{n+1} : \alpha\tau(e_n, e_{n+1}) \wedge \ldots \wedge subj(e_{n+1}, \langle subj\rangle) \wedge \langle dim_v\rangle^{\sigma_v}[e_n, e_{n+1}] \wedge$$
$$\langle dim_p\rangle^{\sigma_p}[e_n, e_{n+1}] \wedge obj^p(e_{n+1}, \langle obj^p\rangle)$$

- Transitive verb, e.g.: *the physician heals the patient*

$$\langle subj\rangle\langle verb_t^{\sigma}(a,t)\rangle\langle obj\rangle \mapsto$$
$$\exists e_{n+1} : \alpha\tau(e_n, e_{n+1}) \wedge \ldots \wedge subj(e_{n+1}, \langle subj\rangle) \wedge$$
$$obj(e_{n+1}, \langle obj\rangle) \wedge \langle dim_v\rangle^{\sigma_v}[e_n, e_{n+1}].$$

Summary

⇒ We thus have derived the conventional syntax of *predicate expressions* demonstrating that these can be considered internally complex constructions.

- We gain the advantage of reducing the number of axioms required for basic and compound transitive relations, such as the temporal *starts*
- and have shown that CL does not replace conventional predicate logic but adds a way to further analyze and better understand its atomic formulae.

⇒ An analogous semantics for CLA, CL0, and CL1 is an analogous semantics for FOL!

Focus and Filter

- We can implement a CL reasoner using a Vector Symbolic Architecture by interpreting vectors logically with bitwise logical operations $\&, |, !$
  - Assume random binary vectors a and b of long length, e.g., 10000 bits
  - Implement $a \sqcap b$ with bitwise &
  - ⇒ Yields a focus operation: focus on those parts of a that are also in b (or those parts of b that are in a)
  - Implement $\sim a$ with bitwise !
  - ⇒ Yields a filter operation: $!a\&b$: filter out parts from b that are in a
  - Implement $a \sqcup b$ with bitwise $|$
  - Implement $a \sqsubseteq b$ as: for all positions i, $a_i \leq b_i$ (or equivalently: for all positions i, $a_i\&!b_i = 0$)

⇒ VSA reasoner is a probabilistic model checker for a classical set-theoretical semantics for CLA, e.g.:
  - ⇒ Given random vectors $n, a, b, c, \ldots$ and a knowledge base (KB) $\phi = n : [a \sqsubseteq b] \ \wedge \ n : [b \sqsubseteq c] \wedge \ldots$
  - Obtain $\phi = !(a\&n\&!b)\&!(b\&n\&!c)\& \ldots$ as the vector encoding of the KB.
  - We can query, e.g., for $n : [a \sqsubseteq c]$ by asking for the encoding of the query $q = !(a\&n\&!c)$, whether $!\phi|q$ is the 1-vector.

From Binary Vectors to Analogous Representations

- We get: $\phi\&x$ as focus on information ϕ has about object/relation x.
- And: $\phi\&n\&a$ as the information about a with respect to the n-relation.
- For the KB $\phi =!(a\&n\&!b)\&!(b\&n\&!c)\&\ldots$ and looking at vectors $\phi\&n\&a$, $\phi\&n\&b$, $\phi\&n\&c$, we see that with ϕ we remove – i.e., set to 0 – all positions i where $\phi_i\&n_i\&a_i = 1$ and $b_i = 0$ and all portions where $\phi_i\&n_i\&b_i = 1$ and $c_i = 0$. In other words, all positions, where $\phi_i\&n_i\&c_i = 1$ have $\phi_i\&n_i\&b_i = 1$ and $\phi_i\&n_i\&a_i = 1$, and all positions, where $\phi_i\&n_i\&b_i = 1$ have $\phi_i\&n_i\&a_i = 1$.

⇒ This means for the number of 1s that $|\phi\&n\&a| \geq |\phi\&n\&b| \geq |\phi\&n\&c|$.

⇒ Generalizing, $|\phi\&n\&x|$ the number of 1s in $\phi\&n\&x$ numerically represents the n-aspect of x, an n-coordinate

⇒ A lot more to say here, e.g.:
  - how to reduce interference between relations (filter out other relations in the query, 2018, 2021)
  - how to rotate/mirror images (2019), etc.
  - how to build CL0 and CL1 reasoning on top of this (conventional FOL Tableaux reasoning, each branch yields a separate image, 2021)

Summary and Conclusions

- Origin of the gap in conventional FOL semantics' (Pierce, Frege) in focus on general relations
- Context Logic is a mature logical language with a unique image semantics
- FOL can be understood as partitioned by CL
- The Activation Bit Vector Machine (ABVM), a logical VSA, provides conventional classical reasoning as well as complex formula grounding and grounded reasoning for CL and thus also FOL
- System can be "shocked" by extreme risk in "game of chicken" scenario

⇒ Demo shows scales in preference dimensions, e.g., wealth, versus options that are maximally catastrophic and tied to existential threat

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