Structural Alignment as an Abductive Integer Linear Programming Problem

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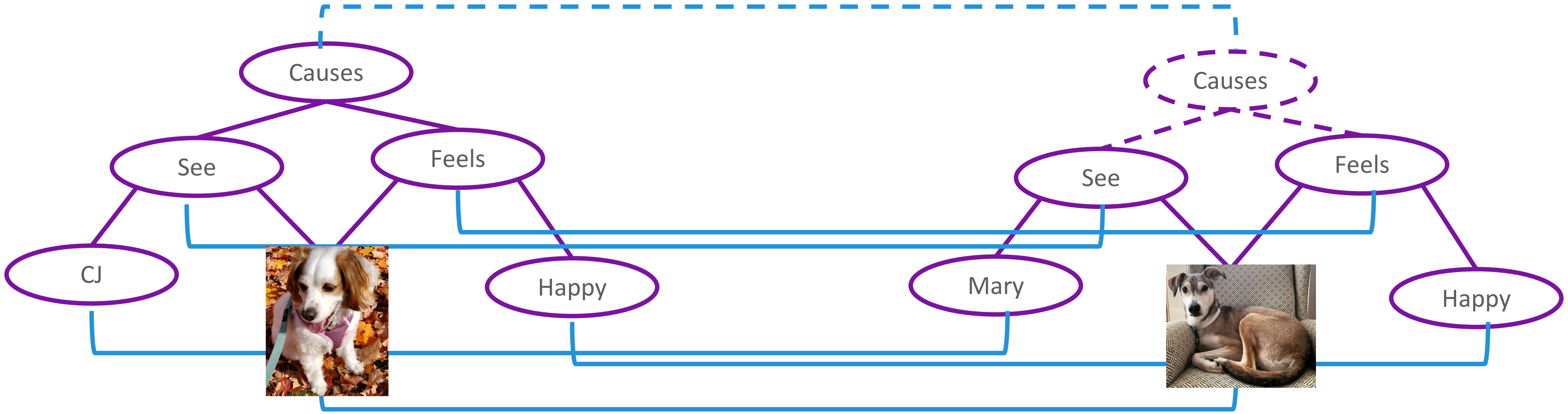
Analogy: Comparing Models

- Analogy plays an important role in many cognitive processes, and computational models of analogy have had interdisciplinary value.
- Cognitive models of analogy often treat analogical reasoning as the alignment of structured representations subject to a set of constraints
    - Historically, there has been debate around what constraints exist, how strong they are, and how they interact with other cognitive processes



Analogy: Comparing Models

	SME	ACME
Approach	Symbolic: • Constraints restrict initial local matches and guide a greedy search for a global mapping	Connectionist • Constraints are edges in a network that excite or inhibit local matches between same arity concepts to create a stable global mapping
Constraints	Hard Constraints: • Identicality • 1-1 Mapping • Parallel Connectivity Soft Constraints: • Systematicity	Hard Constraints: • Identical arity Soft Constraints: • Isomorphism (1-1/parallelism) • Similarity • **Pragmatic Constraints/Goals**
Inferences	Projected from shared structure	Only produced with explicit goals

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CAMMA: A Framework for Experimentation

- A common framework that allows hard structural constraints (like SME) as well as soft biases (like ACME), and that generates testable SME-like inferences, would facilitate hypothesis testing

- Constrained Abductive Mapping Model of Analogy (CAMMA)
  - Formulate analogy as a weighted abductive inference problem
    - Explain two sets of observations (base and target) by their alignment to each other.
  - Represent as a 0-1 Integer Linear Programming problem (Inoue & Inui, 2011)
  - Allows declarative hard and soft constraints on the ILP solve
  - Inferences reduce the cost of the solve by explaining alignable unmapped structures.

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  - Allows declarative hard and soft constraints on the ILP solve
  - Inferences reduce the cost of the solve by explaining alignable unmapped structures.
- More an emulator than a model?
  - CAMMA is not a process-level cognitive model
  - CAMMA lets us see what an optimized alignment looks like under varying theoretical constraints

Weighted Abduction

- Abductive inference seeks the best explanation for an observation
- **Axioms: what makes a road unsafe?**
  - $10: A road is unsafe if it is wet and obscured
  - $10: A road is wet if it has snowed
  - $10: A road is obscured if it has snowed
- **Observation: I95 is unsafe!**
- **Resolve the hypothesis space**
  - I95 could be assumed to be unsafe
  - I95 could be unsafe if we assume it is wet and obscured
  - I95 could be wet if we assume it snowed
  - I95 could be obscured if we assume it snowed
  - Assuming the same snow could cause both conditions; we can pay its cost once, through unification

	unsafe(I95)	wet(I95)	obscured(I95)	snow(I95)	snow(I95)	Cost
H1	assm					40
H2	true	assm	assm			20
H3	true	true	assm	assm		20
H4	true	true	assm	assm		20
H5	true	assm	true		assm	20
H6	true	true	true	assm	assm	20
H7	true	true	true	UNIFIED (assm)		10

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Weighted Abduction As ILP (Inoue & Inui, 2011)

- ILP: Assign optimal integer values to variables given constraints
  - (0,1) can be used to define Boolean properties
    - h{0, 1} 1 if a hypothesis is a part of a solution
    - r{0, 1}: 1 if cost of elemental hypothesis is NOT paid
    - up,q{0, 1}: 1 if elemental hypotheses p and q unify
  - Declarative constraints turn into constraints on values

- H1: Assume I95 is unsafe
- H2: Assume I95 is wet and obscured (pay cost for both)
- H7: Snow caused it all!

	h_u	r_u	h_w	r_w	h_o	r_0	h_{s1}	r_{s1}	h_{s2}	r_{s2}	$U_{s1,s2}$	Cost
H1	1	0	0	1	0	1	0	1	0	1	0	40
H2	1	1	1	0	1	0	0	1	0	1	0	20
H3												
H4												
H5							. . .					
H6												
H7	1	1	1	1	1	1	1	1	1	0	1	10

Analogy as Weighted Abduction

Given two sets of expressions (observations), a **base** and a **target**

1. Each expression can be assumed (left unmapped): $40

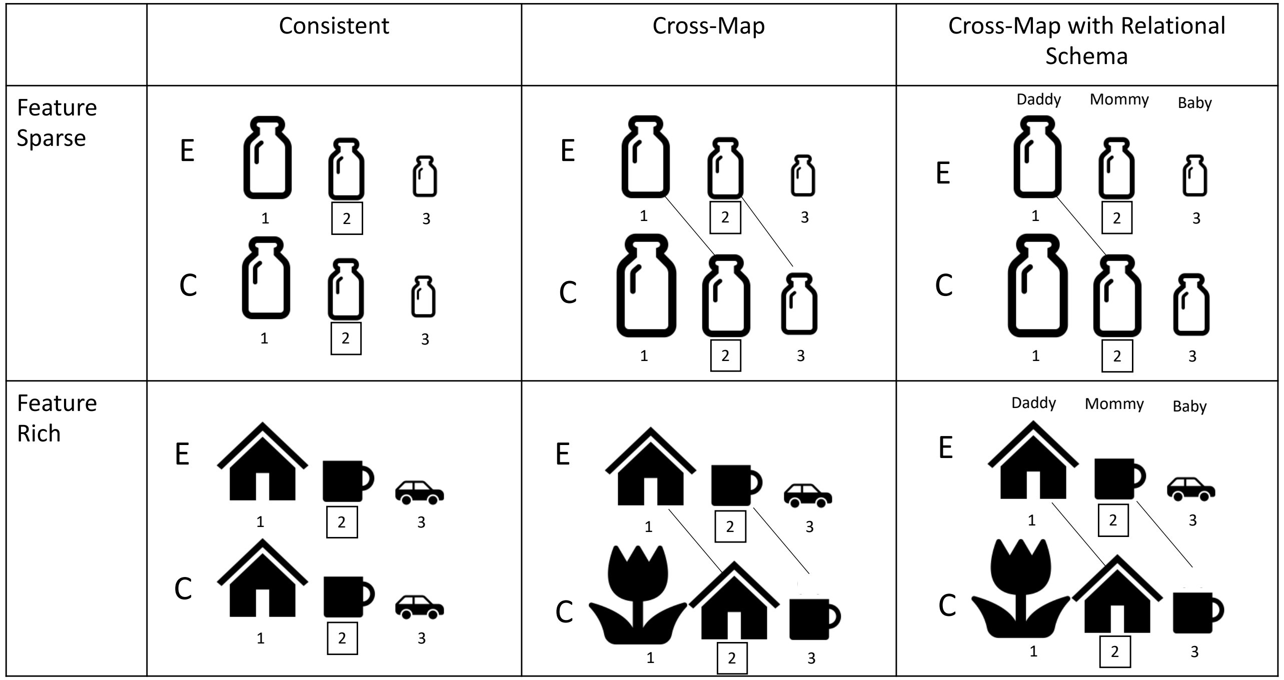
2. For each pair of expressions satisfying identicality $\mathtt{MH}(b_n , t_n)$:
    1. Justify the base and target expressions if is is true that they align:
        - $b_n \texttt{ <= } \mathtt{exprAlign}(b_n , t_n) \texttt{ : } \$20$
        - $t_n \texttt{ <= } \mathtt{exprAlign}(b_n , t_n) \texttt{ : } \$20$
    2. Justify that the expressions align if their children align
        - $\mathtt{exprAlign}(b(x_1 \ldots x_n) , t(x_1 \ldots x_n)) \texttt{ <= } \mathtt{exprAlign}(x_1 \ldots x_n , x_1 \ldots x_n) \texttt{ : } \$20$
        - $\mathtt{exprAlign}(b(x_1 \ldots x_n) , t(x_1 \ldots x_n)) \texttt{ <= } \mathtt{entityAlign}(x_1 \ldots x_n , x_1 \ldots x_n) \texttt{ : } \$1$

3. Expressions can be justified by a CI at a higher cost
    1. A CI justifies an expression by entity alignment of its constituents to a variable
    2. This will be resolved by unification
        - $p(x_1 \ldots x_n) \texttt{ <= } \mathtt{CI}(?x_1 \ldots ?x_n) \texttt{ : } \$5$
        - $\mathtt{CI}(?x_1 \ldots ?x_n) \texttt{ <= } \mathtt{entityAlign}(?x_1 \ldots ?x_n , p(x_1 \ldots x_n)) \texttt{ : } \$1$

How could we use CAMMA to validate cognitive hypotheses?

- Young children seem to prefer matches object/attribute similarity, and develop sensitivity to relational similarity as they mature
    - Is this a cognitive bias and/or could it be explained by acquisition of richer relational schema?
    - How would hard (SME) constraints vs soft (ACME) constraints effect potential matches?

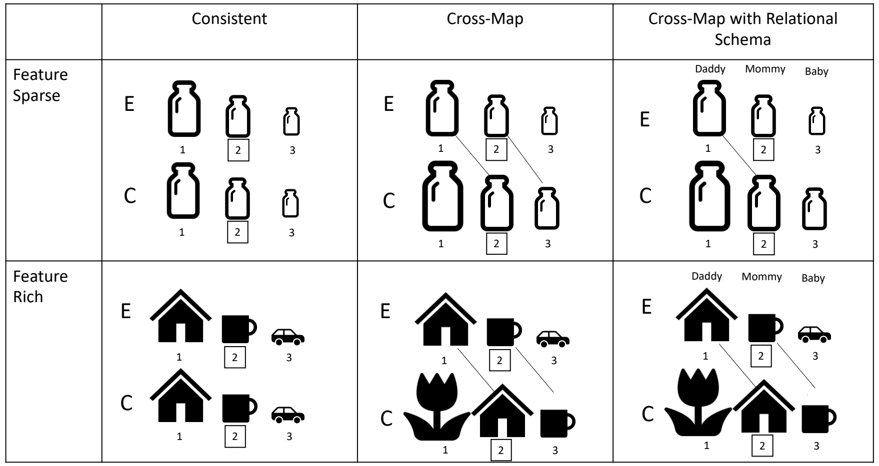
Modeling Relational Shift (Gentner & Ratterman, 1991)



- Test with a bias towards mapping attributes (+ $20 to assume attributes)
- Test with a bias towards relations (+ $20 per unmapped argument)
- Test with hard and soft 1-1 (infeasible solution vs added cost)

Modeling Relational Shift (Gentner & Ratterman, 1991)

- Without a schema, the relation bias had to be increased 3X for a relational match in the Feature Rich X-Map condition
- A relational schema ameliorated the need for a bias, but could be overcome by rich stimuli and/or an attribute bias
- CAMMA predicts that a relational schema facilitates relational mapping, and that this effect is relative to the strength of cognitive bias and feature complexity



Results Without Relational Schema

	1-1 Hard	1-1 Soft
Consistent: None	Object2	Object2
Consistent: Attribute	Object2	Object2
Consistent: Relation	Object2	Object2
Cross-Map: None	Object1	Object3*
Cross-Map: Attribute	Object1	Object3*
Cross-Map: Relation	Object2	Object3*
Feature Rich Cross-Map: None	Object1	Object1
Feature Rich Cross-Map: Attribute	Object1	Object1
Feature Rich Cross-Map: Relation	Object1	Object1
Feature Rich Cross-Map: 3xRelation	Object2	Object1

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Feature Rich Cross-Map: Relation	Object2	Object2*
Feature Rich Cross-Map: 3xRelation	Object2	Object2*

Conclusions and Future Work

- CAMMA formulates analogy as an abductive reasoning ILP problem
- While not a process-level cognitive model, CAMMA facilitates exploration of different mapping constraints as well as the impact of cognitive biases
- CAMMA could incorporate analogy into a general abductive ILP problem solver
    - Possible analogical cases could be retrieved and used to explain observations via their alignment in isolation, conjunction, or competition with other paths to a viable solve

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